Shadowing in Deuterium.

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Abstract

We calculate nuclear shadowing in lepton-deuteron deep inelastic scattering, which arises from the double scattering of the virtual photon from both nucleons in the deuteron. The total correction to the deuteron structure function is found to be $\lesssim 1\%$ at small x, but dependent on the model deuteron wavefunction. The resulting increase in the corrected neutron structure function is $\sim 1-2\%$ for $x \simeq 0.004$, which leads to a 4-10% decrease in the value of the Gottfried sum obtained recently by the New Muon Collaboration.

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1 Introduction

The quark structure of the nucleon is one of the most fundamental aspects of hadron physics. Deep inelastic scattering (DIS) of leptons from hydrogen has yielded a wealth of information on the deep inelastic structure of the proton. However, the absence of free neutron targets has forced one to use deuterium in order to extract data on the neutron structure functions. Traditionally in DIS on the deuteron, in which the proton and neutron are held together very weakly, nuclear effects have been ignored, and the total lepton-deuteron cross section assumed to be the sum of the lepton-proton and lepton-neutron cross sections. It is the deviation from this simple relation in the region of small Bjorken x ($x \lesssim 0.1$) which is known as shadowing.

Experimentally, a deviation from linearity has been observed [1] in the so-called nuclear EMC effect for the ratio of DIS cross sections (or structure functions) for lepton scattering from a nucleus and from deuterium. A dramatic decrease in the nuclear structure function per nucleon in the region of small x confirmed earlier predictions [2] that shadowing should be present in DIS. Furthermore, the shadowing was found to be only weakly dependent on Q^2 . The extraction of information about the difference between nuclear structure functions and those for the free nucleon from the observed nucleus/deuterium ratios is sensitive to any nuclear effects in the deuteron. Conclusions made about nucleon parton distributions based on the nuclear/deuteron structure function ratios (eg. for the proton antiquark distributions in the Drell-Yan process [3]) at small x may have to be modified once shadowing is taken into account.

A precise knowledge of the neutron structure function, F_{2n} , is essential for the determination of the Gottfried sum rule, and the corresponding resolution of the question of flavour symmetry violation in the proton sea. It is necessary therefore to check for nuclear shadowing effects in deuterium and include this correction in the extraction of F_{2n} from the deuteron structure function, F_{2D} . Some recent estimates [4, 5] have suggested a significant amount of shadowing in deuterium (up to 4%) for $x \lesssim 0.1$. Other calculations [6] have predicted a less dramatic effect ($\approx 2\%$).

The cross section for lepton-deuteron DIS, fig.1, is related to the forward γ^*D scattering amplitude.

In the impulse approximation, fig.2, the virtual photon interacts with one of the nucleons in the nucleus. The double scattering diagram, fig.3, in which both nucleons participate in the interaction, is the origin of the shadowing in a nucleus.

2 Vector Meson Dominance

Hadron-Deuteron Glauber Scattering

Glauber theory [7, 8] for hadron-deuteron scattering gives the total hD cross section as a sum of the hN cross sections, and a screening term arising from the double scattering of both nucleons:

$$\sigma_{hD} = 2\sigma_{hN} + \delta\sigma_{hD} \tag{1}$$

where

$$\delta\sigma_{hD} = -\frac{\sigma_{hN}^2}{8\pi^2} \int d^2\mathbf{k}_T S_D(\mathbf{k}^2)$$

$$= -\frac{\sigma_{hN}^2}{4\pi} \int dk \ k S_D(\mathbf{k}^2), \tag{2}$$

with $k \equiv |\mathbf{k}|$. In deriving $\delta \sigma_{hD}$, the assumption is made that the hadron-nucleon scattering amplitude, \mathcal{F}_{hN} , is primarily imaginary, $\text{Re}\mathcal{F}_{hN} \ll \text{Im}\mathcal{F}_{hN}$, and approximately independent of \mathbf{k}^2 for small \mathbf{k}^2 . (Contributions to $\delta \sigma_{hD}$ from large \mathbf{k}^2 will be suppressed by the deuteron form factor $S_D(\mathbf{k}^2)$.) Then from the forward double scattering amplitude [9]

$$\delta \mathcal{F}_{hD} = \frac{i}{2\pi |\mathbf{q}|} \int d^2 \mathbf{k}_T \ S_D(\mathbf{k}^2) \ \mathcal{F}_{hp}(\mathbf{k}^2) \ \mathcal{F}_{hn}(\mathbf{k}^2), \tag{3}$$

where \mathbf{q} is the momentum of the projectile, eqn. (2) follows via the optical theorem:

$$\sigma = \frac{4\pi}{|\mathbf{q}|} \, \mathrm{Im} \mathcal{F}.$$

γ^*D Scattering

Assuming that the Glauber formalism can be applied to γ^*D scattering, the shadowing correction to the γ^*D cross section was originally calculated in terms of the vector meson dominance (VMD) model,

where the virtual photon dissociates into its hadronic components (vector mesons) before interacting with the nucleon – see fig.4. In this model the shadowing cross section is given by [10]

$$\delta^{(V)}\sigma_{\gamma^*D} = \sum_{v} \frac{e^2}{f_v^2} \frac{1}{(1+Q^2/M_v^2)^2} \delta\sigma_{vD}$$
 (4)

where $v = \rho^0, \omega, \phi$, and the photon-vector meson coupling constants are [11]

$$\frac{f_v^2}{4\pi} = \frac{\alpha^2 M_v}{3 \Gamma_{v \to e^+ e^-}} \tag{5}$$

(equal to 2.28, 26.14, 14.91 for ρ^0 , ω and ϕ , respectively ¹). Writing (4) in terms of the deuteron structure function, F_{2D} ², we have

$$\delta^{(V)}F_{2D}(x) = \frac{Q^2}{\pi} \sum_{v} \frac{\delta \sigma_{vD}}{f_v^2 (1 + Q^2/M_v^2)^2},$$
 (6)

where now

$$\delta\sigma_{vD} = -\frac{\sigma_{vN}^2}{8\pi^2} \int d^2\mathbf{k}_T \ S_D(\mathbf{k}^2). \tag{7}$$

The total vector meson–nucleon cross sections, σ_{vN} , are related to the total πN and KN cross sections via the quark model, and are set to 24 mb for $v = \rho^0$ and ω , and 14.5 mb for $v = \phi$ (see [10, 12]). The deuteron form factor $S_D(\mathbf{k}^2)$ is given by the electric monopole body form factor [13]

$$S_D(\mathbf{k}^2) = \int_0^\infty dr \left(u^2(r) + w^2(r) \right) j_0(kr), \tag{8}$$

²In terms of the total cross section for the photo-absorption of virtual photons on an unpolarised deuteron, σ_{γ^*D} , the deuteron structure function is

$$W_{2D} = \frac{K}{4\pi^2 \alpha} \frac{Q^2}{Q^2 + \nu^2} \sigma_{\gamma^* D}$$

where $K = \sqrt{\nu^2 + Q^2}$ is the flux of incoming virtual photons (in the Gilman convention), so that in the Bjorken limit

$$F_{2D} = \frac{Q^2}{4\pi^2 \alpha} \sigma_{\gamma^* D}.$$

¹Note that the fine structure constant evaluated at $Q^2 = \mathcal{O}(1\text{GeV}^2)$ is $\alpha = e^2/4\pi \approx 1/130$, although the error introduced by this is probably less than that associated with using f_v^2 , which is obtained from the decay of meson v with time-like Q^2 , for the coupling to a photon with space-like Q^2 .

where u(r), w(r) are the S, D-wave deuteron wavefunctions, normalised s.t. $\int dr \ (u^2(r) + w^2(r)) = 1$, and where j_0 is the spherical Bessel function. The square of the 3-momentum transfer to the interacting nucleon is $\mathbf{k}^2 = \mathbf{k}_T^2 + k_L^2$, where $k_L^2 = m_N^2 x^2 (1 + M_v^2/Q^2)^2$, and $x = Q^2/2p \cdot q$.

From eqn.(6) it can be seen that the VMD shadowing correction to the deuteron structure function decreases as $1/Q^2$ for $Q^2 \to \infty$.

At $Q^2 = 4 \text{GeV}^2$ the VMD model shadowing predictions are given in fig.5 for deuteron form factors obtained from several different NN potential models. By far the largest contribution ($\approx 80\%$) to $\delta^{(V)} F_{2D}$ comes from the ρ^0 meson. The magnitude of $\delta^{(V)} F_{2D}(x)$ decreases with x because the lower limit of the k-integration in eqn.(7), $k_{min} = k_L$, is an increasing function of x, and the integrand peaks at small values of $k \approx 0.7 \text{ fm}^{-1}$. The model dependence arises from differences in the large- $k \approx 2 \text{ fm}^{-1}$ behaviour of the form factor, fig.6, which itself is determined by the small-r behaviour of u(r), w(r). All of the deuteron wavefunctions obtained from realistic NN potential models (namely Paris [14], Bonn (OBEPQ) [15] and Bochum [16]) produce a trough in $kS_D(\mathbf{k}^2)$ at $k \approx 3.5 \text{ fm}^{-1}$ (because the Bessel function is negative at large kr), and a rapid fall-off with k for $k \approx 6 \text{ fm}^{-1}$. Also shown is the model of Franco and Varma [17], which was used in [4, 5], for which the form factor, parameterised by a sum of Gaussians, has no large-k tail at all. The form factor with the Paris wavefunction, which has the 'deepest' trough, leads to $\delta^{(V)}F_{2D}$ which is $\approx 25\%$ smaller for $x \approx 0.01$ than with the Franco and Varma form factor. The trough is also responsible for the antishadowing in the region $x \approx 0.2$.

3 Diffractive Scattering from Partons

At low Q^2 , it is most natural to evaluate the γ^*D shadowing in terms of the VMD model. At higher energies a parton picture may be more relevant. An alternative description of the double interaction mechanism in fig.3 in the high energy limit is in terms of Pomeron (\mathcal{P}) exchange, fig.7. If the momentum transfer between the photon and nucleon is small, the nucleon will most likely remain intact, in which case there will only be exchange of vacuum quantum numbers. Although

there is as yet no QCD-based derivation of the properties of the reactions described by Pomeron exchange (eg. constant hadronic cross sections), there have been suggestions [18, 19, 20] that the Pomeron represents a system of gluons. (In ref.[18] hadron-hadron scattering is modelled in terms of gluon exchange between MIT bags, while in ref.[20] gluon-ladder techniques are used to calculate deep inelastic structure functions of hadrons at low x.)

In fig.7 the virtual photon probes the parton structure of the Pomeron, which is parameterised by the Pomeron structure function F_{2P} [21, 22] (defined in terms of the cross section for γ^* -Pomeron diffractive scattering):

$$F_{2\mathcal{P}}(x) \equiv \frac{Q^2}{4\pi^2 \alpha} \sigma_{\gamma^* \mathcal{P}}. \tag{9}$$

The contribution to the F_{2D} structure function from multiple diffractive scattering with \mathcal{P} exchange can be written as a convolution of an exchange- \mathcal{P} distribution function, $f_{\mathcal{P}}(y)$, with the \mathcal{P} structure function:

$$\delta^{(P)} F_{2D}(x) = \int_{y_{min}}^{2} dy \ f_{P}(y) \ F_{2P}(x_{P})$$
 (10)

where

$$f_{\mathcal{P}}(y) = -\frac{\sigma_{pp}}{8\pi^2} \frac{1}{y} \int d^2 \mathbf{k}_T S_D(\mathbf{k}^2)$$
 (11)

is expressed as a function of the momentum fraction of the nucleon carried by the Pomeron, $y = k \cdot q/p \cdot q = x(1 + M_X^2/Q^2) \approx M_X^2/s$ $(M_X^2 = p_X^2, s = (p+q)^2)$, and we define $x_P \equiv x/y$. Fig.8 illustrates the y-dependence of $f_P(y)$, including the 1/y divergence for $y \to 0$. The rapid fall off with y is testament to the very small contribution coming from the large-y region.

In formulating a complete description of shadowing which includes more than one mechanism care must be taken to avoid possible double counting. Because of this concern some authors [6] have restricted the Pomeron exchange process to the region of M_X^2 above the highest mass of the vector mesons contributing to the VMD process: $M_X^2 \geq M_{X_0}^2 \simeq 1.5 \text{GeV}^2$, and consequently have taken the lower bound on the integral in eqn.(10) to be $y_{min} = x(1 + M_{X_0}^2/Q^2)$. The VMD contribution, which is essentially a higher twist $(1/Q^2)$ effect, may compete with that part of the diagram in fig.7 which

contains low- M_X single particle intermediate states. By keeping only the leading twist piece of the structure function $F_{2\mathcal{P}}$, we can exclude this contribution since it involves extra factors of $1/Q^2$ from the electromagnetic form factors. Nevertheless, we have tested the sensitivity of our numerical results to the cut-off procedure by varying $M_{X_0}^2$ from 0 to 2 GeV². For low x we find a difference over this range of only some 5% of the total \mathcal{P} exchange contribution to F_{2D} . For larger Q^2 the separation into separate M_X regions becomes irrelevant since $y_{min} \to x$ in the Bjorken limit.

For the Pomeron structure function we include contributions from the quark-antiquark box diagram, fig.9a, and from the triple Pomeron interaction, fig.9b (see refs.[23, 24]):

$$F_{2P}(x_P) = F_{2P}^{(box)}(x_P) + F_{2P}^{(3P)}(x_P)$$
 (12)

normalised such that

$$F_{2\mathcal{P}} = \left(\frac{16\pi y}{\sigma_{pp}}\right) \frac{d^2 F_2^{diff}}{dt \ dy}\bigg|_{t=0}, \tag{13}$$

where $t \approx -\mathbf{k}^2$, and F_2^{diff} is the diffractive structure function, describing semi-inclusive diffractive lepton-nucleon DIS, in which the recoil nucleon and the hadronic state X are separated by a large rapidity [22].

The Pomeron structure function arising from the quark box diagram, $F_{2P}^{(box)}$, has been calculated by Donnachie and Landshoff [22]:

$$F_{2P}^{(box)}(x_P) = \frac{(12\Sigma_{q^2} N_{sea}) \beta_0^2}{\sigma_{pp}} x_P (1 - x_P).$$
 (14)

The quark—Pomeron coupling constant is $\beta_0^2 = 3.4 \text{GeV}^{-2}$ [25], and we assume the same strength for u, d quark and antiquark—Pomeron couplings, but a weaker coupling to the strange quarks: $\Sigma_{q^2} = (10 + 2\lambda_s)/9$ with $\lambda_s \simeq 0.5$. According to the Particle Data Group [11], the proton-proton total cross section σ_{pp} is approximately 40 mb. The parameter N_{sea} is determined by the $x \to 0$ behaviour of the nucleon sea distribution, $xq_{sea}(x \to 0) \to N_{sea}x^a$. Recent parameterisations of world DIS, Drell-Yan and prompt photon data [26, 27, 28] give $N_{sea} \simeq 0.15$, and a approximately 0. Note that the overall normalisation of the r.h.s. of eqn.(14) is slightly smaller than in [6] due to our smaller sea parameter N_{sea} (cf. $N_{sea} = 0.17$ in [6]) and suppression of strange—Pomeron couplings. More recently, Nikolaev

and Zakharov [24] have calculated the box diagram contribution to $F_{2\mathcal{P}}$, based on a perturbative QCD analysis of $q\bar{q}$ fluctuations of the virtual photon. The $x_{\mathcal{P}}$ dependence of their $F_{2\mathcal{P}}^{(box)}$ parameterisation is the same as that in eqn.(14): $M_X^2/(Q^2+M_X^2)^3$ (since $Q^2+M_X^2=Q^2/x_{\mathcal{P}}$ from the definition of $x_{\mathcal{P}}$), providing the same normalisation is used (the normalisations in [22] and [24, 29] differ by an overall factor $1-x_{\mathcal{P}}$).

The triple Pomeron part of the \mathcal{P} structure function,

$$F_{2\mathcal{P}}^{(3\mathcal{P})}(x_{\mathcal{P}}) = \frac{16\pi}{\sigma_{pp}} \left[\frac{y}{\sigma_{hp}} \left. \frac{d^2 \sigma_{hp \to hX}}{dt dy} \right|_{t=0} \right] F_{2N}^{sea}(x_{\mathcal{P}}, Q^2)$$
(15)

follows from

$$\frac{1}{F_{2N}^{sea}} \left. \frac{d^2 F_2^{diff}}{dt dy} \right|_{t=0} = \left. \frac{1}{\sigma_{hp}} \left. \frac{d^2 \sigma_{hp \to hX}}{dt dy} \right|_{t=0} \tag{16}$$

and the Regge theory expression for the diffractive differential cross section [30]

$$\frac{d^2 \sigma_{hp \to hX}}{dt du} = \frac{\beta_{h\mathcal{P}}(t) \beta_{p\mathcal{P}}^2(t) g_{3\mathcal{P}}(t)}{16\pi} y^{1 - 2\alpha_{\mathcal{P}}(t)}$$
(17)

where $\alpha_{\mathcal{P}}(t) \approx 1 + 0.25t$. In the Regge model the total hp cross section is also given in terms of the hadron—Pomeron couplings, $\beta_{h\mathcal{P}}$: $\sigma_{hp} = \beta_{h\mathcal{P}}(0)\beta_{p\mathcal{P}}(0)$. It is then evident that the combination

$$\left. \frac{1}{\sigma_{hp}} \frac{d^2 \sigma_{hp \to hX}}{dt dy} \right|_{t=0} = \frac{\beta_{p\mathcal{P}}(0) \ g_{3\mathcal{P}}(0)}{16\pi y} \tag{18}$$

is independent of hadron h. From experiments on the diffractive dissociation of π^{\pm} , K^{\pm} , p and \bar{p} on hydrogen, the triple Pomeron coupling constant was found to be $g_{3\mathcal{P}}(0) \simeq 0.364 \text{ mb}^{1/2}$ [31], independent of t, and indeed of the hadron type h.

For the sea part of the nucleon structure function, $F_{2N}^{sea} = 5x(u_s + \bar{u} + d_s + \bar{d} + 2(s + \bar{s})/5)/18$, we use recent parameterisations of the data at $Q^2 = 4 \text{GeV}^2$ [27, 28]. In the calculation of ref.[6], a constant value of 0.3 was used for F_{2N}^{sea} together with an empirical low- Q^2 dependence [22]. With the above triple Pomeron coupling constant, eqn.(15) gives a $3\mathcal{P}$ component which is about 40% smaller than that obtained in [4]. However, this is not very significant for the total Pomeron structure function, since $F_{2\mathcal{P}}^{(3\mathcal{P})}$ is very much smaller than the quark-antiquark 'box' contribution, $F_{2\mathcal{P}}^{(box)}$, fig.10.

The scaling behaviour of the \mathcal{P} -exchange mechanism is determined by the scaling behaviour of the \mathcal{P} structure function, and from eqns.(14)—(18) it is clear that $\delta^{(\mathcal{P})}F_{2D}$ will scale as $Q^2 \to \infty$. At $Q^2 = 4\text{GeV}^2$, fig.11a shows the individual 'box' and $3\mathcal{P}$ contributions to $\delta^{(\mathcal{P})}F_{2D}$, with the deuteron form factor obtained from the Bochum wavefunction. The dependence of $\delta^{(\mathcal{P})}F_{2D}$ on $S_D(\mathbf{k}^2)$ is illustrated in fig.11b. Again, as in the case of the VMD model, the large-k negative tail of the form factor produces a large (some 30-40%) difference between different models for $x \lesssim 0.05$. For $x \gtrsim 0.2$ the presence or absence of antishadowing will be determined by the model deuteron wavefunction.

4 Shadowing by Mesons

Another potential source of shadowing arising from the double scattering mechanism is one which involves the exchange of mesons, fig.12. It has previously been suggested [32] that this leads to substantial antishadowing corrections to $F_{2D}(x)$. The total contribution to the deuteron structure function from meson exchange is written

$$\delta^{(M)} F_{2D}(x) = \sum_{\mu} \int_{x}^{m_D/m_N} dy \ f_{\mu}(y) \ F_{2\mu}(x_{\mu}), \tag{19}$$

where $\mu = \pi, \rho, \omega, \sigma$ and $y = k \cdot q/p \cdot q = (k_0 + k_L)/m_N$ and $x_\mu = x/y$. For the virtual meson structure function, $F_{2\mu}$, we take the parameterisation of the (real) pion structure function from Drell-Yan production [33]. The exchange-meson distribution functions $f_\mu(y)$ are obtained from the non-relativistic reduction of the nucleon—meson interaction:

$$f_{\mu}(y) = 4c_{\mu} \ m_{N} \int \frac{d^{3}\mathbf{p} \ d^{3}\mathbf{p}'}{(2\pi)^{3}} \frac{\mathcal{F}_{\mu NN}^{2}(k^{2})}{(k^{2} - m_{\mu}^{2})^{2}} \ y \ \left\{ \frac{1}{3} \sum_{J_{z}} \Psi^{\dagger}(\mathbf{p}, J_{z}) \ \mathcal{V}_{\mu NN} \ \Psi(\mathbf{p}', J_{z}) \right\} \delta\left(y - \frac{k_{0} + k_{L}}{m_{N}}\right).$$
 (20)

The deuteron wavefunction is defined by

$$\Psi(\mathbf{p}, J_z) = \frac{1}{\sqrt{4\pi}} \left(u(p) - w(p) \frac{S_{12}(\hat{p})}{\sqrt{8}} \right) \chi_1^{J_z}, \tag{21}$$

where u(p) and w(p) are its S and D-wave components, normalised so that $\int dp \, \mathbf{p}^2 \, (u^2(p) + w^2(p)) = 1$, with $\hat{p} \equiv \mathbf{p}/p$ and $p \equiv |\mathbf{p}|$, and S_{12} is the tensor operator: $S_{12}(\hat{p}) = 3 \, \sigma_1 \cdot \hat{p} \, \sigma_1 \cdot \hat{p} - \sigma_1 \cdot \sigma_2$. The deuteron spin wavefunction is denoted by $\chi_1^{J_z}$, where J_z is the total angular momentum projection.

In eqn.(20), $k^2 = k_0^2 - \mathbf{k}^2$, where $k_0 = m_D - \sqrt{m_N^2 + \mathbf{p}^2} - \sqrt{m_N^2 + \mathbf{p}'^2}$ is the energy of the off-shell meson, and $\mathbf{k} = \mathbf{p} - \mathbf{p}'$ is its 3-momentum.

The nucleon—meson interactions are given by [15]

$$\mathcal{V}_{\pi NN} = -\frac{f_{\pi NN}^2}{m_{\pi}^2} \, \sigma_1 \cdot \mathbf{k} \, \sigma_2 \cdot \mathbf{k} \tag{22}$$

$$\mathcal{V}_{\rho NN} = g_{\rho NN}^2 \left[1 + \frac{3\mathbf{q}^2}{2m_N^2} - \frac{\mathbf{k}^2}{8m_N^2} - \sigma_1 \cdot \sigma_2 \frac{\mathbf{k}^2}{4m_N^2} + \frac{\sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}}{4m_N^2} \right]
+ \frac{g_{\rho NN} f_{\rho NN}}{2m_N} \left[-\frac{\mathbf{k}^2}{m_N} - \sigma_1 \cdot \sigma_2 \frac{\mathbf{k}^2}{m_N} + \frac{\sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}}{m_N} \right]$$
(23)

$$+ \quad \frac{f_{\rho NN}^2}{4m_N^2} \left[-\sigma_1 \cdot \sigma_2 \ \mathbf{k}^2 + \sigma_1 \cdot \mathbf{k} \ \sigma_2 \cdot \mathbf{k} \right]$$

$$\mathcal{V}_{\omega NN} = g_{\omega NN}^2 \left[1 + \frac{3\mathbf{q}^2}{2m_N^2} - \frac{\mathbf{k}^2}{8m_N^2} - \sigma_1 \cdot \sigma_2 \frac{\mathbf{k}^2}{4m_N^2} + \frac{\sigma_1 \cdot \mathbf{k} \sigma_2 \cdot \mathbf{k}}{4m_N^2} \right]$$
(24)

$$\mathcal{V}_{\sigma NN} = -g_{\sigma NN}^2 \left[1 - \frac{\mathbf{q}^2}{2m_N^2} + \frac{\mathbf{k}^2}{8m_N^2} \right], \tag{25}$$

where $\mathbf{q} = \frac{1}{2}(\mathbf{p} + \mathbf{p}')$. Terms proportional to $\mathbf{S} \cdot \mathbf{k} \times \mathbf{q}$, where $\mathbf{S} = \sigma_1 + \sigma_2$, are omitted as they do not contribute to $f_{\mu}(y)$.

Evaluation of eqn.(20) requires the identities:

$$\frac{1}{3} \sum_{J_z} \Psi^{\dagger}(\mathbf{p}, J_z) \ \Psi(\mathbf{p}', J_z) = \frac{1}{4\pi} \left[u(p) \ u(p') + w(p) \ w(p') \ P_2(\cos \theta) \ P_2(\cos \theta') \right]
+ \phi \text{ dependent terms}$$

$$= \frac{1}{3} \sum_{J_z} \Psi^{\dagger}(\mathbf{p}, J_z) \ \sigma_1 \cdot \sigma_2 \ \Psi(\mathbf{p}', J_z)$$
(26)

$$\frac{1}{3} \sum_{J_z} \Psi^{\dagger}(\mathbf{p}, J_z) \, \sigma_1 \cdot \mathbf{k} \, \sigma_2 \cdot \mathbf{k} \, \Psi(\mathbf{p}', J_z) = \frac{1}{4\pi} \left\{ \frac{1}{3} \left[\mathbf{k}^2 - 2 \, p \, p' \, \sin \theta \, \sin \theta' \, \right] u(p) \, u(p') \right. \\
\left. - \frac{1}{\sqrt{2}} \left[4 \, p \, p' \, \cos \theta \, \cos \theta' \, \sin^2 \theta' \, + \, 4 \, \mathbf{p}'^2 \cos^2 \theta' \, \sin^2 \theta' \right. \\
\left. - \frac{2}{3} \left(\mathbf{p}^2 + \mathbf{p}'^2 \right) \, P_2(\cos \theta') \, + \, 2 \left(\mathbf{p}^2 \cos^2 \theta + \mathbf{p}'^2 \cos^2 \theta' \right) \, P_2(\cos \theta') \right. \\
\left. + \frac{8}{3} \, p \, p' \, \cos \theta \, \cos \theta' \, P_2(\cos \theta') \right] \, u(p) \, w(p') \\
- \frac{1}{\sqrt{2}} \left[4 \, p' \, p \, \cos \theta' \, \cos \theta \, \sin^2 \theta + \, 4 \, \mathbf{p}^2 \cos^2 \theta \, \sin^2 \theta \right. \\
\left. - \frac{2}{3} \left(\mathbf{p}'^2 + \mathbf{p}^2 \right) \, P_2(\cos \theta) \, + \, 2 \left(\mathbf{p}'^2 \cos^2 \theta' + \mathbf{p}^2 \cos^2 \theta \right) \, P_2(\cos \theta) \right. \\
\left. + \frac{8}{3} \, p' \, p \, \cos \theta' \, \cos \theta \, P_2(\cos \theta) \right] \, w(p) \, u(p')$$

$$-\frac{1}{3}\left[(p\cos\theta+p'\cos\theta')^{2}P_{2}(\cos\theta)P_{2}(\cos\theta')\right]$$

$$-2\left(\mathbf{p}^{2}\sin^{2}\theta+\mathbf{p'}^{2}\sin^{2}\theta'\right)P_{2}(\cos\theta)P_{2}(\cos\theta')$$

$$+3\left(\mathbf{p}^{2}\cos^{2}\theta\sin^{2}\theta+p\ p'\cos\theta\cos\theta'\sin^{2}\theta\right)P_{2}(\cos\theta')$$

$$+3\left(\mathbf{p'}^{2}\cos^{2}\theta'\sin^{2}\theta'+p'\ p\cos\theta'\cos\theta'\sin^{2}\theta'\right)P_{2}(\cos\theta')$$

$$+\frac{9}{2}p\ p'\cos\theta\cos\theta'\sin^{2}\theta'\sin^{2}\theta\sin^{2}\theta'\right]w(p)\ w(p')$$

$$+\phi\ dependent\ terms.$$
(27)

The terms in eqns.(26)-(27) which depend on the azimuthal angle (ϕ) vanish after integration. The factors c_M are due to isospin: $c_{\pi} = c_{\rho} = 3$, $c_{\omega} = c_{\sigma} = -1$. The μNN vertex form factors $\mathcal{F}_{\mu NN}(k^2)$ are parameterised by a dipole form

$$\mathcal{F}_{\mu NN}(k^2) = \left(\frac{\Lambda_{\mu}^2 - m_{\mu}^2}{\Lambda_{\mu}^2 - k^2}\right)^2, \tag{28}$$

with the high-momentum cut-offs Λ_{μ} ranging from $\sim 1 \text{GeV}$ in models with soft form factors [34, 16] to $\sim 1.7 - 2 \text{GeV}$ when hard form factors are employed [15].

Fig.13 shows the individual meson exchange contributions to $\delta^{(M)}F_{2D}$, for the wavefunction of the Bonn model, and with a universal dipole cut-off of $\Lambda_{\mu}=1.7 {\rm GeV}$. As could be expected, pion exchange is the dominant process. We also include the fictitious σ meson, but with a mass (≈ 800 MeV) that is larger than that used to represent 2π exchange in NN scattering. Both of these produce antishadowing for small x. The exchange of vector mesons (ρ, ω) cancels some of this antishadowing, although the magnitude of these contributions is smaller. In fact, for $\Lambda_{\mu} \stackrel{<}{\sim} 1.3$ GeV all contributions other than that of the pion are totally negligible.

Fig.14 shows the dependence of the total $\delta^{(M)}F_{2D}$ on Λ_{μ} for the Bonn model wavefunction. There is approximately a factor of 2 difference between the amount of shadowing with soft ($\Lambda_{\mu} = 1 \text{GeV}$, lower dotted line) and hard ($\Lambda_{\mu} = 1.7 \text{GeV}$, upper dotted line) form factors. In lepton-nucleon DIS it is well known [35] that the meson cloud of the nucleon, with a hard μNN form factor, gives nucleon sea distributions that are several times larger than the empirical ones. In fact, to be consistent with the lepton-nucleon DIS data Λ_{μ} must be $\approx 0.8 - 0.9 \text{GeV}$. We also consider the effect of the

model momentum-space deuteron wavefunction on $\delta^{(M)}F_{2D}$. Although the model wavefunctions differ substantially at large momenta ($p \gtrsim 2 \text{ fm}^{-1}$), this variation will be largely suppressed by the μNN form factor. The Bochum and Paris wavefunctions are generally larger than the Bonn wavefunction, and this is reflected in a larger $\delta^{(M)}F_{2D}$.

We also comment here on the issue raised in the previous section, namely double counting, this time between the meson exchange and the other mechanisms. It should be clear that since the \mathcal{P} contribution involves the exchange of vacuum quantum numbers, there will be no interference between this and the exchange of pseudoscalar pions or vector mesons. The scalar σ meson, introduced as an effective description of two-pion $N\Delta$ excitations, does not correspond to actual exchange of a spin 0 particle. By restricting the meson structure function to only the leading twist component (our F_{2M} is determined at $Q^2 = 25 \text{ GeV}^2$ where this assumption is reasonable) we may view the VMD process as a description of higher twist effects. Still, imposing any low- M_X cut on the meson exchange term has numerically insignificant consequences, largely because $F_{2\mu}(x/y) \to 0$ as $y \to x$.

5 Combined Shadowing Effects and the Gottfried Sum Rule

The total deuteron structure function is defined by

$$F_{2D}(x) = F_{2p}(x) + F_{2n}(x) + \delta F_{2D}(x),$$
 (29)

where the shadowing correction is a sum of the VMD, Pomeron and meson exchange contributions:

$$\delta F_{2D}(x) = \delta^{(V)} F_{2D}(x) + \delta^{(P)} F_{2D}(x) + \delta^{(M)} F_{2D}(x).$$
 (30)

In fig.15 we compare the contributions to $\delta F_{2D}(x)$ from the three mechanisms considered. For $x \lesssim 0.1$ the magnitude of the (negative) Pomeron/VMD shadowing is larger than the (positive) meson-exchange contribution, so that the total δF_{2D} is negative. The fact that shadowing is present in this region of x does not depend on the model deuteron wavefunction. For larger $x \approx 0.1 - 0.2$ there is a small amount of antishadowing, which is due mainly to the VMD contribution. The dependence of the total shadowing correction on the deuteron wavefunction and on the μNN form factor is shown

in fig.16 for $Q^2 = 4 \text{ GeV}^2$. We point out that the magnitude of $\delta F_{2D}(x)$ is about 4 times smaller than that obtained in reference [4], and about 2 times smaller compared with the result of reference [6]. The most important reasons for our smaller results are the inclusion of meson exchange contributions which produce antishadowing at small x, and the use of realistic deuteron wavefunctions which lead to smaller \mathcal{P} exchange and VMD contributions.

Recently the New Muon Collaboration (NMC) has measured F_{2p} and F_{2D} [36, 37] down to very small values of x (= x_{min} = 0.004). The neutron structure function was then extracted from F_{2D} in order to test the Gottfried sum rule [38]. However, by assuming that

$$\frac{F_{2D} \left(1 - (F_{2D}/F_{2p} - 1)\right)}{\left(1 + (F_{2D}/F_{2p} - 1)\right)} = 2F_{2p} - F_{2D} \equiv (F_{2p} - F_{2n})_{NMC}$$
(31)

the NMC ignored any nuclear shadowing effects in D which may alter the F_{2n} values. The actual difference between the p and n structure functions should be

$$F_{2p} - F_{2n} = (F_{2p} - F_{2n})_{NMC} + \delta F_{2D}, \tag{32}$$

and this is shown in fig.17. The dotted line is a best fit to the NMC data, and includes the small-x extrapolation used in [37]:

$$F_{2n}(x) - F_{2n}(x) \xrightarrow{x \to 0} \alpha x^{\beta}$$
 (33)

with $\alpha = 0.21, \beta = 0.62$. The other curves include the shadowing corrections to the NMC data parameterisation. It is not clear whether $F_{2p} - F_{2n}$ will become negative at $x \stackrel{<}{\sim} 0.004$, and it will be interesting to see whether this cross-over occurs when additional data at smaller x become available.

The Gottfried integral

$$S_G(x,1) = \int_x^1 dx' \frac{F_{2p}(x') - F_{2n}(x')}{x'}$$

$$= \int_{x'=x}^1 d \left(\log x' \right) \left(F_{2p}(x') - F_{2n}(x') \right)$$
(34)

is given in fig.18 for x down to 0.004. In the naive quark model, $S_G(0,1) = 1/3$. Ignoring nuclear effects, the NMC obtained $S_G(x_{min}, 1) = 0.229$. From the unmeasured region (x < 0.004), using the above extrapolation, the contribution was found to be $S_G(0, x_{min}) = (\alpha/\beta) x_{min}^{\beta} = 0.011$. With the

Model	α	β	$S_G(0, x_{min})$	$S_G(x_{min}, 1)$	$S_G(0,1)$
NMC [37]	0.21	0.62	0.011	0.229	0.240 ± 0.016
	0.109	0.5	0.014		0.243
Bochum ($\Lambda_{\mu} = 1.3 \text{GeV}$)	0.043	0.5	0.005	0.222	0.227
Paris $(\Lambda_{\mu} = 1.3 \text{GeV})$	0.052	0.5	0.007	0.224	0.230
Bonn $(\Lambda_{\mu} = 1.3 \text{GeV})$	0.011	0.5	0.001	0.215	0.217
Bonn $(\Lambda_{\mu} = 1.0 \text{GeV})$	0.002	0.5	0.000	0.214	0.214
Bonn $(\Lambda_{\mu} = 1.7 \text{GeV})$	0.019	0.5	0.002	0.217	0.219

Table 1: Small-x extrapolation parameters for $F_{2p} - F_{2n} (= \alpha x^{\beta})$ and the contributions to the Gottfried sum from different x-regions.

conventional Regge theory assumption that $\beta = 0.5$, $S_G(0, x_{min})$ would be 0.014. In table 1 we give the values of S_G including shadowing corrections, and also the $x < x_{min}$ extrapolation parameters. For simplicity we take $\beta = 0.5$, and adjust α to achieve a smooth transition between the $x > x_{min}$ and $x < x_{min}$ regions. The overall correction to the NMC value for $S_G(0,1)$ is found to be between -0.010 and -0.026. This is to be compared with -0.07 to -0.088 obtained in [4, 5, 29].

As a fraction of the total $F_{2D}(x)$ [37], the shadowing correction amounts to (0.5-1.0%, 0.4-0.8%, 0.0-0.3%) at x = (0.004, 0.01, 0.1), while the antishadowing is less than 0.2% of F_{2D} at $x \approx 0.2$.

In fig.19 we show the ratio of neutron structure functions with and without shadowing corrections,

$$\frac{F_{2n}}{(F_{2n})_{NMC}} = 1 - \frac{\delta F_{2D}}{F_{2D}} \left(\frac{1 + (F_{2n}/F_{2p})_{NMC}}{(F_{2n}/F_{2p})_{NMC}} \right)$$
(35)

where the NMC neutron/proton ratio was defined as $(F_{2n}/F_{2p})_{NMC} \equiv F_{2D}/F_{2p} - 1$. There is an overall 1-2% increase in the neutron structure function due to shadowing for $x \lesssim 0.01$.

Finally, we illustrate in fig.20 the dependence upon Q^2 of the total shadowing correction, $\delta F_{2D}(x,Q^2)$. As expected, the VMD term vanishes rapidly with increasing Q^2 , leaving the two scaling contributions from \mathcal{P} and meson exchange to largely cancel each other for $Q^2 \simeq 25 \text{ GeV}^2$. However, we should add a note of caution about comparing shadowing corrections at very large values of Q^2 . In the parton recombination model [2, 39, 40] the fusion of quarks and gluons from different nucleons introduces additional terms [39] in the Altarelli-Parisi equations governing the QCD evolution of the parton distributions. At very small x and large Q^2 , such as those attainable at HERA energies, this can lead to significant corrections [6] to the $\delta F_{2D}(x,Q^2)$ evolved without these terms, although the exact magnitude of these is sensitive to the small-x behaviour of the input nucleon gluon distribution. For the moderate range of Q^2 and not too low x values in fig.20, however, we expect the indicated Q^2 behaviour to be reliable.

6 Conclusion

In summary, we have estimated the nuclear shadowing in lepton-deuteron DIS from the double scattering mechanism in fig.3. Our approach is similar to that of refs.[5] and [6], in describing the interaction in terms of the VMD model, together with Pomeron (\mathcal{P}) exchange at larger M_X . However we have also included contributions from the exchange of mesons which effectively cancel as much as half of the shadowing from the VMD/ \mathcal{P} -exchange mechanisms alone. Numerically, there is some dependence on the model deuteron wavefunction, and also on the meson–nucleon form factor for the meson-exchange process. The net effect is a $\approx 1\%$ reduction of F_{2D} for $x \sim 0.004$, or equivalently a $\approx 2\%$ increase in the neutron structure function over the uncorrected F_{2n} . Consequently, the shadowing correction to the Gottfried sum $S_G(0,1)$ is between -0.010 and -0.026 (or about 4 and 10% of the NMC value), which is about 5 times smaller than in previous estimates.

To accurately test the descriptions of shadowing in the deuteron it is necessary to obtain model-independent information on the neutron structure function at low x. Even at HERA energies this is not possible with electron scattering alone. However, when combined with high-precision data from neutrino-proton experiments the individual flavour distributions can be determined, and the neutron structure function inferred from charge symmetry. For this to happen, however, the statistics on the

neutrino data need to be improved, and the range extended to smaller x.

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Figure captions

- 1. Lepton-deuteron deep inelastic scattering.
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- 4. Double scattering mechanism in the vector meson dominance model. The virtual photon dissociates into a vector meson which then scatters from the nucleon.
- 5. VMD contribution to the total deuteron structure function at $Q^2 = 4 \text{ GeV}^2$, for different model deuteron form factors.
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- 14. Deuteron wavefunction and μNN form factor dependence of the total meson exchange correction. The Bochum (solid) and Paris (dashed) curves are evaluated with $\Lambda_{\mu} = 1.3$ GeV, while the Bonn (dotted) curves have $\Lambda_{\mu} = 1.0, 1.3$ and 1.7 GeV, with the larger cut-off giving more overall antishadowing.
- 15. Comparison between the VMD, Pomeron and meson exchange corrections to the deuteron structure function at $Q^2 = 4 \text{ GeV}^2$ (for the Bochum wavefunction, and a form factor cut-off $\Lambda_{\mu} = 1.3$ GeV for the meson exchange process).
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